An Integrated Two-warehouse Deteriorating Inventory Model with Shortage backordering , Trade Credit and Decreasing Warehouse Rental

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ABSTRACT

This study develops an optimal inventory model for items with deteriorating loss and shortage backordering in a two-echelon supply chain system. It consists of one supplier and one distributor with a two-warehouse environment where the storage price of the rented warehouse decreases over time. Owners of rented warehouses can decrease the storage price when a certain time is reached as an incentive to distributors. In addition, offering a credit period stimulates suppliers' selling and reduces on-hand stock levels. Furthermore, distributor can use this credit period to reduce costs and increase profits. This study determines the optimal production lot size of both players and the number of shipments to minimize the total cost. In addition, it demonstrates that an optimal solution exists and is unique. A numerical example and sensitivity analyses are provided to illustrate the proposed model. The results of this study provide managerial insight for enterprises that use a rented warehouse to minimize costs by coordinating lot sizes.

***Keywords*:** Two-warehouse; Deteriorating items; Inventory; Shortage backordering; Trade credit

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1. INTRODUCTION

Due to rising costs, decreasing resources, shortening product life cycles and quicker response time, most companies around the world are realizing that internal efficiency is not enough to compete in today's global markets. Through collaboration, different facilities develop their partnership to achieve long-term benefits and global optimality of the system. Integration of material processes within and between companies is one of the most fundamental trends in business these days (Chikán, 2007).

Many studies have investigated multi-echelon inventory and distribution systems. The integration of supply chains significantly increases the benefits of the entire system. Bessler and Veinott (1966) performed an analysis on the integration between buyers and suppliers by developing a mathematical model with an arborescent structure. Goyal (1976) considered an integrated inventory model for the single-supplier single-customer problem. The approach of joint economic lot size (JELS) is an efficient method for integration in supply chains and has been a topic of study for years. One of the advantages of using JELS is its ability to generate lower total inventory relevant costs, enabling the net benefits to be shared by both parties. Banerjee (1985) developed JELS with a lot-for-lot policy for a single-buyer single-vendor system by combining two EOQ models from the buyer and the vendor. In his model, Banerjee assumed that vendors perform production setup if buyers place orders and require supplies on a lot-for-lot basis. By considering both buyers and the vendors simultaneously, Banerjee showed that the JELS model has a minimum joint total relevant cost.

Now-a-days, in real practice, suppliers/wholesalers usually provide credit period to encourage larger orders from customers who benefit from the delay payment. Here, credit period is treated as a promotional tool as it is one kind of price discount because paying later circuitously reduces the purchase cost and motivates retailers to increase the ordered quantity. In the last two decades, the inventory models with trade credit have been widely studied by several researchers. Haley and Higgins (1973) were the first to consider trade credit financing in inventory research. Goyal (1985) was the first who established an EOQ model with a constant demand rate under the condition of permissible delay in payments. Aggarwal and Jaggi (1995) then extended Goyal's (1985) model for deteriorating items. They developed ordering policies for deteriorating items under permissible delay in payments. Ongoing deteriorating inventory has been studied by several researchers in recent decades. Ghare and Schrader (1963) were the first authors to consider the ongoing deterioration of inventory, and they developed an EOQ model for items with an exponentially decaying inventory. Min et al. (2010) developed an inventory model for deteriorating items under stock-dependent demand and two-level trade credit. Deterioration of units is one of the most crucial factors in inventory problems for deteriorating items. There are several articles on the inventory of deteriorating items with considering trade credit financing in inventory research such as Bakker (2012), Shah (2015) etc. Ouyang et al. (2009) developed two inventory models for deteriorating items with permissible delay in payment. Some notable research papers on deteriorating items incorporating various types of assumptions are due to He et al. (2010), Hung (2011), Sana (2011), etc. Most of the above inventory models are developed with constant deterioration. But deterioration also increases with time as stress of units on others causes damage. Also due to nature of these items rate of deterioration increases with time. According to the author's best knowledge, very few articles have been published incorporating time varying deterioration Sarkar (2011). Dye (2012) presented a finite time horizon deteriorating inventory model and solved it using Particle Swarm Optimization (PSO). An overall progress of inventory research for deteriorating item has been presented in the review article of Bakker et al. (2012). Das et al. (2013) developed an integrated supply chain model under trade credit policy.

Two-warehouse inventory concept for merchants is an age-old one. This is followed to avoid frequent transportation inconvenience, to avail the advantage of price concession, to guard the scarcity of the commodity, etc. Now-a-days, the two-warehouse inventory system has become more important due to prevailing volatile marketing condition and stiff competitions among the gradually increasing national and international sellers. Initially most of the inventory models assumed that every manufacturer have its own warehouse with unlimited capacity but in reality we know that every warehouse have limited capacity. Enterprises usually use the two-warehouse system, enabling enterprises to buy large quantities of items, thereby taking advantage of price discounts for bulk purchase and reserving an alternative warehouse in case the ordered quantities exceed the capacity of the first warehouse. A rented warehouse (RW) is used when the capacity of an enterprise’s own warehouse (OW) is insufficient to accommodate all of their storage items. RW may have better preservative performance due to its good condition, low risk factor, low deterioration rate, and better storage facility. But manufacturer have to pay some rent or cost for that which is much greater than OW. Although it is typically assumed that alternative warehouses offer better facilities, this requires a higher inventory cost. So that inventory managers use the RW but try to utilize the inventory of RW first and then OW inventory is used. Therefore, the two-warehouse storage strategy is a critical decision-making activity for enterprises with supply chains in reducing costs.

Some research has already been performed in this field of study. Hartely (1976) was the first to consider the effects of a two-warehouse system model in an inventory model with an RW storage policy. Sarma (1983) developed a two-warehouse model with constant demand. In addition, Sarma (1987) developed a two-warehouse model for deteriorating items with an infinite replenishment rate and shortage. Pakkala and Achary (1991) presented a two-warehouse probabilistic order-level inventory model for deteriorating items. Pakkala and Achary (1992) further examined the two-warehouse model for deteriorating items with an finite replenishment rate and shortage. Furthermore, Pakkala and Achary (1992) developed a discrete-in-time model for deteriorating items with two warehouses. Ishii and Nose (1996) investigated the optimal ordering policies for a perishable product with various types of customer priorities, selling prices, and OW capacity constraints. Benkherouf (1997) extended Sarma's model and relaxed the assumptions of a fixed cycle length and a specified quantity to that has to be stocked in OW. Bhunia and Maity (1998) analyzed a deterministic inventory model with linearly increasing demand, shortages, and various levels of item deterioration for both warehouses. Zhou and Yang (2005) developed a deterministic inventory model with an inventory-dependent demand rate and two separate warehouses, OW, and RW. They assumed that the demand rate was a polynomial form of the current inventory level and that the stock is transferred to the OW in a bulk release pattern with transportation costs. Yang (2012) proposed a two-warehouse deterioration inventory model with partial backlogging under inflation. Recently Shah & Cárdenas-Barrón (2015) proposed the retailer's decision for ordering and credit policies is analyzed when a supplier offers its retailer either a cash discount or a fixed credit period. Recently, Majumdera *et al.* (2016) presented an integrated production inventory model of supplier and retailer where a delay in payment is accessible by supplier towards the retailer and also by retailer en route for customer.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model was based on the following assumptions:

(1) The planning horizon is finite and composed of several equal periods.

(2) Single item, one order, and multi-delivery are assumed.

(3) The demand rate is constant and known.

(4) The production rate is deterministic.

(5) The deteriorated items will cause shortages.

(6) Shortage is allowed and completely backordered.

(7) The replenishment of the distributor is instantaneous. The related transportation time can be neglected.

(8) There is a limited storage capacity for the distributor.

(9) To reduce inventory costs, it is economical to consume the goods of RW as early as possible. Consequently, firms store goods in the OW before they do in the RW, but they clear the stock in the RW before the OW.

(10) t_d is the distributor's trade credit period offered by the supplier in a year. When $t \geq t_d$, the account is settled at time $t = t_d$ and the distributor begins paying for the interest charges on the items in stock with rate i_c. In this study, there are three conditions for t_d: $t_1 \geq t_d$, $t_1 \leq t_d \leq t_1 + t_2$, and $t_1 \leq t_d$.

The following notations are used in this study:

q	lot size per delivery from the supplier to the distributor
n	number of shipments delivered from the supplier to the distributor during the planning horizon T
C_S	set-up cost per set-up for the supplier ($/set-up)
P_M	cost of deteriorated units for the supplier ($/unit)
H_M	holding cost for the supplier ($/unit/ year)
θ_1	deterioration rate for the supplier
p	production rate (unit/unit time), $p>d$
d	demand rate (unit/unit time)
C_R	replenishment cost per cycle for the distributor ($/cycle)
C_o	the backordering cost per unit
H_{Ro}	holding cost for the distributor in the OW ($/unit/ year)
H_{Rr}	holding cost for the distributor in the RW ($/unit/ year)
P_R	cost of deteriorated units for the distributor ($/unit)
θ_2	deterioration rate in the Rw
θ_3	deterioration rate in the OW
w	available storage capacity in the OW
i_c	interest charged per dollar per year
i_e	interest earned per dollar per year
*	the superscript representing optimal value

3. THE MODEL

This study proposes a deteriorating JELS model for a single-supplier and single-distributor with a two-warehouse environment considering that the storage price of the RW decreases over time. It was assumed that a company owned a warehouse with a fixed capacity and stores excess supplies in an RW with an unlimited capacity. We assumed that the owner of RWs lower their storage prices following a specific storage time to a value less than the OW storage price as an incentive mechanism to extend their partnerships. Enterprises typically store their items in an OW prior to an RW and clear the stock in RWs before they do in OWs. However, following a reasonable storage time, if the storage price of an RW is less than the OW, enterprises may prefer to store their goods in an RW instead of an OW and clear the stock in their OW before clearing their stock in the RW. We assumed a constant demand rate, limited distributor warehouse capacity, and infinite RW capacity. The production rate was finite and shortage was allowed. The study determined the optimal production lot size of the supplier and the distributor to minimize their total costs.

3.1 THE SUPPLIER'S INVENTORY SYSTEM

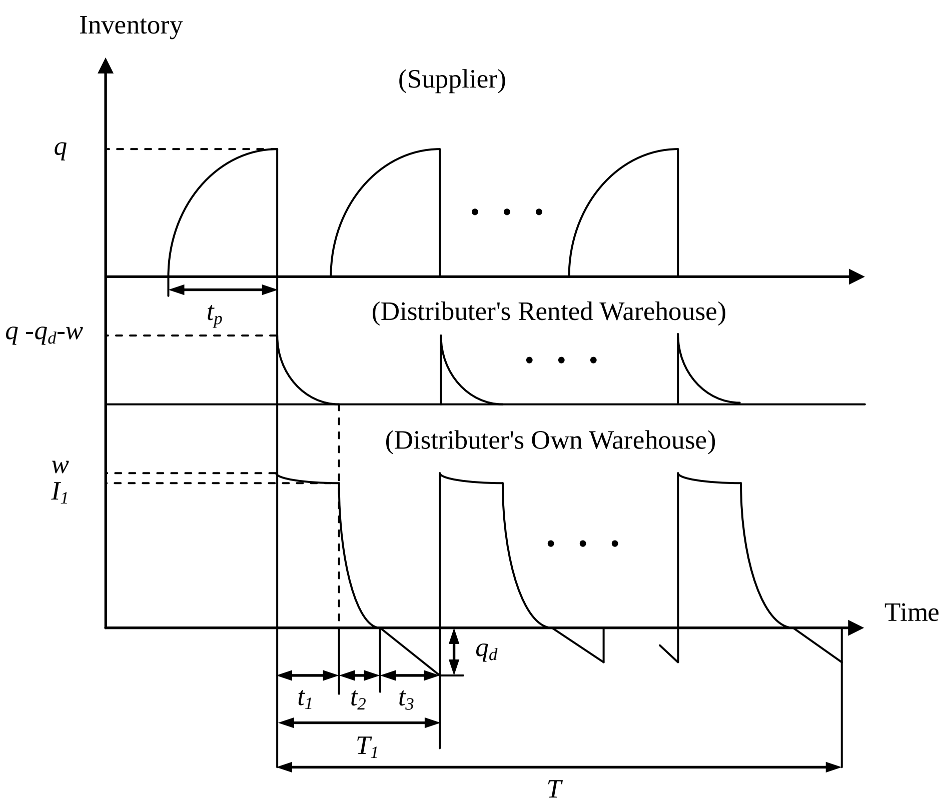

Fig. 1. Inventory level of finished goods

$I_M(t)$ is the supplier's finished goods inventory level at time t. The supplier's inventory system in Fig. 1 is depicted by the following differential equation:

$$\frac{dI_M(t)}{dt} = p - \theta_1 I_M(t) \quad , \quad 0 \le t \le t_p \quad , \quad t_p = T_1 - t_m \tag{1}$$

where t_m is the production lead time and OW is used up at time T_1.
With the various boundary conditions $I_M(0) = 0$ and $I_M(t_p) = q$, one has,

$$I_M(t) = \frac{p}{\theta_1}\left[1 - e^{-\theta_1 t}\right] \quad , \quad 0 \le t \le t_p \tag{2}$$

From (2), the production cycle time is,

$$t_p = t_p(q) = \frac{1}{\theta_1}\ln\left[\frac{p}{p - q\theta_1}\right] \quad \text{and} \quad 0 \le q \cdot \theta_1 < p \tag{3}$$

The total set-up cost during the planning horizon is

$$SC = \sum_{j=1}^{n} C_s = n \times C_s \tag{4}$$

The holding cost during the planning horizon is

$$HC_M = \sum_{j=1}^{n} H_M \int_0^{t_p} I_M(t)dt \tag{5}$$

The corresponding deteriorating quantity is $q_{Md} = pt_p - q$. From (3), one has

$$q_{Md} = \frac{p}{\theta_1}\ln\left[\frac{p}{p - q\theta_1}\right] - q \tag{6}$$

Thus, the total deteriorating cost during the planning horizon can be expressed as

$$DC_M = \sum_{j=1}^{n} P_M \times \left(p\left[\frac{\ln(p)}{\theta_1} - \frac{\ln(p - q\theta_1)}{\theta_1} \right] - q \right) = n \times P_M \times \left(p\left[\frac{\ln(p)}{\theta_1} - \frac{\ln(p - q\theta_1)}{\theta_1} \right] - q \right) \quad (7)$$

The supplier's inventory cost during the planning horizon T, TC_M, is the sum of the set-up cost (SC), the holding cost (HC_M) and the deteriorating cost (DC_M). One has

$$TC_M(q,n) = SC + HC_M + DC_M \quad (8)$$

3.2 THE DISTRIBUTOR'S INVENTORY SYSTEM

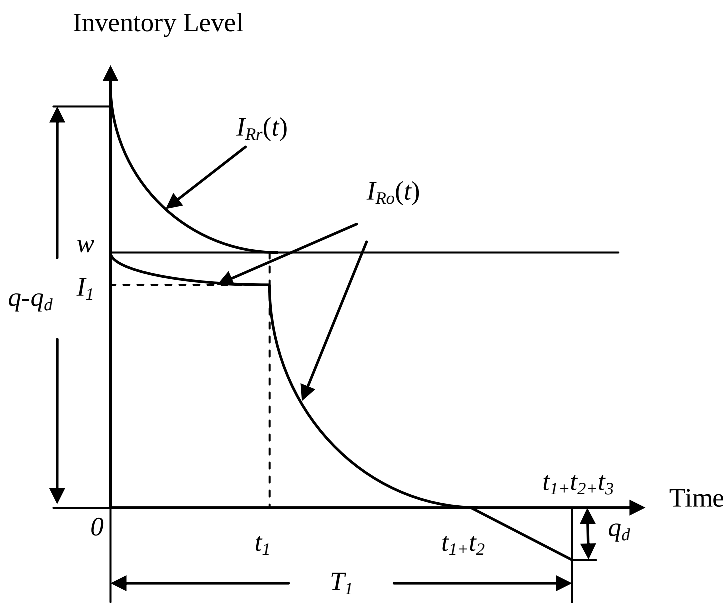

Fig. 2. Inventory level of the distributor with retailer's demand

The distributor's inventory system with OW and RW is depicted in Fig. 2. The inventory level in OW, $dI_{Ro}(t)$, during an infinitesimal time, dt, is a function of the deterioration rate θ_2, the demand rate d and the inventory level $I_{Ro}(t)$. The inventory level in RW, $dI_{Rr}(t)$, during an infinitesimal time, dt, is a function of the deterioration rate θ_3, the demand rate d and the inventory level $I_{Rr}(t)$. In common practice, RW offers an incentive policy if the stock is kept for a longer period of time. The unit holding cost decreases by u% per unit of time; the unit holding cost per unit of time at time t is $h(t)$.

$$h(t_r) = \begin{cases} H_{Rr} e^{-\alpha t_r} & ,0 < t_r \le t_{\max} \\ H_{Rr} e^{-\alpha t_{\max}} , & t_{\max} < t_r \end{cases} \tag{9}$$

where t_r is the tenancy for RW, t_{max} is the longest storage time desired by RW and α is given by $\alpha = -\ln(1-u/100)$.

$I_{Rr}(t)$ is the distributor's inventory level in RW at time t. The change in the inventory level of RW is formulated as,

$$\frac{dI_{Rr}(t)}{dt} = -d - \theta_2 I_{Rr}(t) \quad , \quad 0 \le t \le t_1 \tag{10}$$

where the RW is used up at time t_1.

From the above differential equations, after adjusting for constant integration with the various boundary conditions: $I_{Rr}(0)=q-w$ and $I_{Rr}(t_1)=0$, one has

$$I_{Rr}(t) = \frac{d}{\theta_3}\left[e^{\theta_2(t_1-t)} - 1\right] \quad , \quad 0 \le t \le t_1 \tag{11}$$

The corresponding time is

$$t_1 = \frac{1}{\theta_3}\ln\left[\left(\frac{\theta_2}{d}\right)(q-w)+1\right] \tag{12}$$

$I_{Ro}(t)$ is the distributor's inventory level in OW at time t. The change in the inventory level of OW during an infinitesimal time can be formulated as

$$\frac{dI_{Ro}(t)}{dt} = -\theta_3 I_{Ro}(t) \quad , \quad 0 \le t \le t_1 \tag{13}$$

$$\frac{dI_{Ro}(t)}{dt} = -d - \theta_3 I_{Ro}(t) \quad , \quad t_1 \le t \le t_1 + t_2 \tag{14}$$

From (13) and (14), and with the various boundary conditions $I_{Ro}(0)=W$, $I_{Ro}(t_1)=I_1$ and $I_{Ro}(T_1)=0$, the solution of the above differential equations is

$$I_{Ro}(t)=\begin{cases} we^{-\theta_3\cdot t}, & 0\le t\le t_1 \quad (15)\\ \dfrac{-d+e^{-\theta_3(t-t_1)}\left(d+I_1\theta_3\right)}{\theta_3}, & t_1\le t\le t_1+t_2 \quad (16)\end{cases}$$

The inventory level of OW at t_1 is $I_1=we^{-\theta_3\cdot t_1}$. From (15) and (16), the replenishment cycle is

$$t_2=\frac{1}{\theta_3}\ln\left[1+\frac{I_1\theta_3}{d}\right]=\frac{1}{\theta_3}\ln\left[1+\frac{we^{-\theta_3 t_1}\theta_3}{d}\right] \tag{17}$$

The total ordering cost during the planning horizon is

$$ORC=\sum_{j=1}^{n}C_R=n\times C_R \tag{18}$$

The total holding cost in OW during the planning horizon is

$$HC_R^{\mathrm{Rent}}=\sum_{j=1}^{n}H_R^{\mathrm{Rent}}=n\times\int_0^{t_1}\frac{d}{\theta_2}\left[e^{\theta_2(t_1-t)}-1\right]\cdot H_{Rr}e^{-\alpha t_r}\,dt \tag{19}$$

The total holding cost in RW during the planning horizon is

$$HC_R^{Own}=\sum_{j=1}^{n}H_R^{Own}=n\times H_{Ro}\left(\int_0^{t_1}we^{-\theta_3 t}dt+\int_{t_1}^{t_1+t_2}\left(\frac{-d+e^{-\theta_3(t-t_1)}\left(d+I_1\theta_3\right)}{\theta_3}\right)dt\right) \tag{20}$$

The total deteriorating cost in RW during the planning horizon is

$$DC_R^{\mathrm{Rent}}=P_R\cdot\theta_2\int_0^{t_1}I_{Ro}(t)\cdot dt \tag{21}$$

For the deteriorated items cause shortages, the deteriorated quantity is

$$q_d=\theta_2\int_0^{t_1}I_{Ro}(t)\cdot dt=d\cdot t_3. \tag{22}$$

One has

$$t_3 = \frac{\theta_2}{d}\int_0^{t_1} I_{Ro}(t)\cdot dt \tag{23}$$

Therefore, the total backordering cost during the planning horizon is

$$OC = \sum_{j=1}^{n} C_o \int_0^{t_3} (t\cdot d)dt = n\times\, C_o \int_0^{t_3} (t\cdot d)dt \tag{24}$$

The total deteriorating cost in OW during the planning horizon is

$$DC_R^{Own} = P_R\cdot\theta_3\int_0^{t_1} I_{Rr}(t)\cdot dt + P_R\cdot\theta_3\int_{t_1}^{t_1+t_2} I_{Rr}(t)\cdot dt \tag{25}$$

Thus, one has

$$DC_R = DC_R^{\mathrm{Re}nt} + DC_R^{Own} = \sum_{j=1}^{n} P_R\cdot\left(\theta_2\int_0^{t_1} I_{Rr}(t)dt + \theta_3\int_0^{t_1} I_{Ro}(t)dt + \theta_3\int_{t_1}^{t_1+t_2} I_{Ro}(t)dt\right) \tag{26}$$

According to assumption (10), three cases may occur in calculation of interest charges for the items kept in stock per year.

Case1. $t_1 \geq t_d$ (shown in Fig. 3)

Cost of interest charges per year can be obtained as follow:

$$C_i = i_c\sum_{j=1}^{n}\left[h(t_r)\int_{t_d}^{t_1} I_{Rr}(t)dt + H_{Rr}\left(\int_{t_1}^{t_1+t_2} I_{Ro}(t)dt + \int_0^{t_1} I_{Ro}(t)dt\right)\right] \tag{27}$$

Interest earned per year can be obtained as follow:

$$C_e = i_e\sum_{j=1}^{n} h(t_r)\int_0^{t_d}(d\cdot t)dt \tag{28}$$

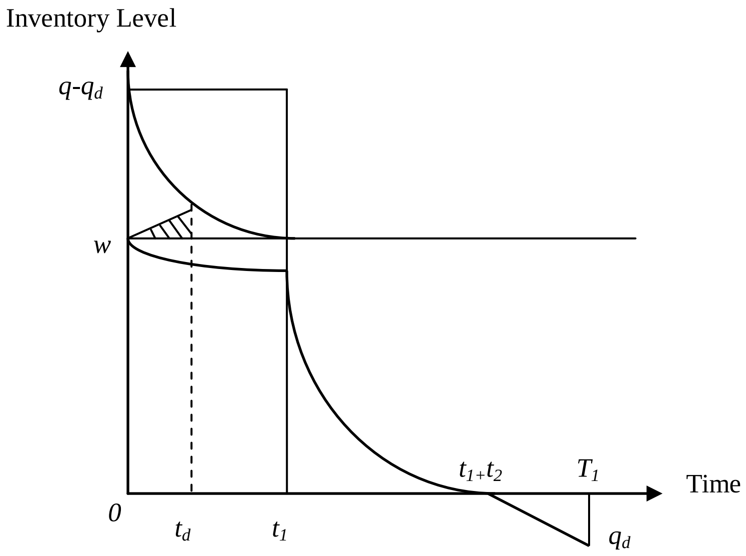

Fig. 3. The total accumulation of interest earned when $t_1 \geq t_d$.

Case2. $t_1 \leq t_d \leq t_1 + t_2$ (shown in Fig. 4)

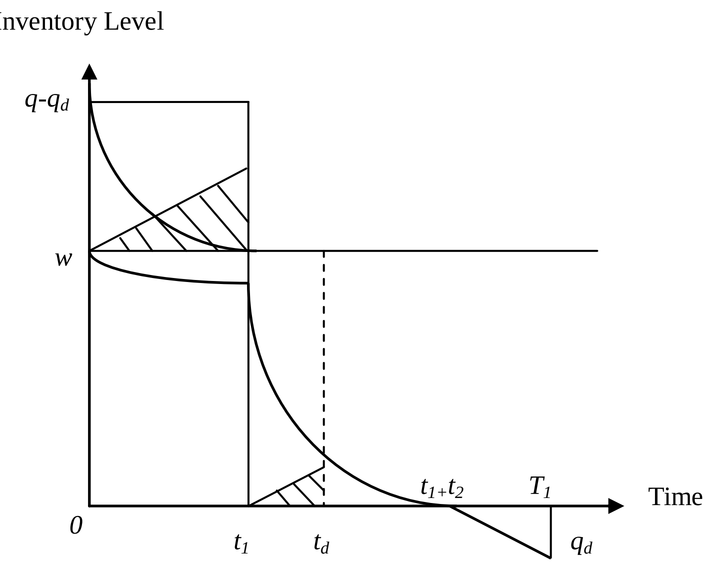

Fig. 4. The total accumulation of interest earned when $t_1 \leq t_d \leq t_1 + t_2$.

Cost of interest charges per year can be obtained as follow:

Interest payable per year $= C_i = i_c \sum_{j=1}^{n} H_{Ro} \int_{t_d}^{t_1+t_2} I_{Ro}(t)dt$ (29)

Interest earned per year can be obtained as follow:

Interest earn per year = $C_e = i_e \sum_{j=1}^{n} \left[h(t_r) \int_0^{t_1} (d \cdot t) dt + H_{Ro} \int_{t_1}^{t_d} (d \cdot t) dt \right]$ (30)

Case3. $t_1 + t_2 \le t_d$ (shown in Fig. 5)

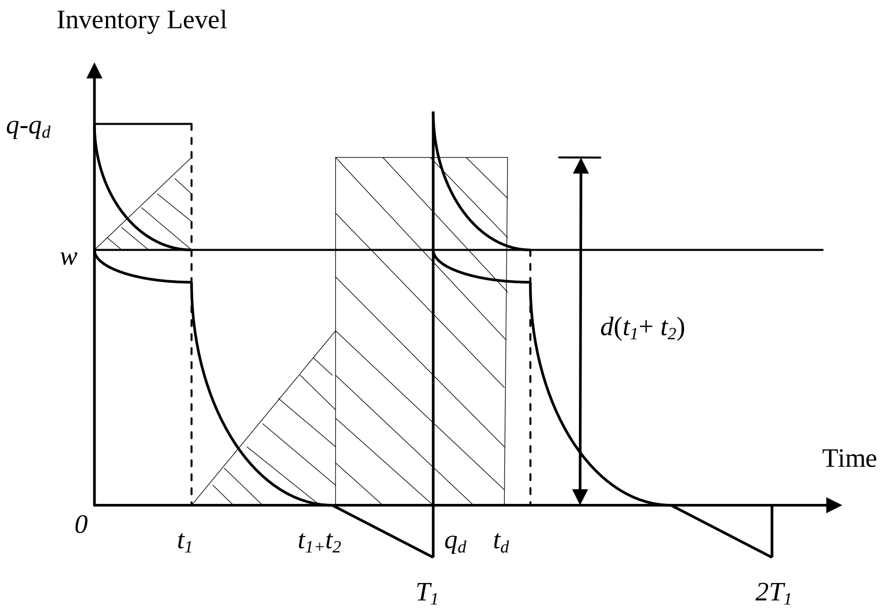

Fig. 5. The total accumulation of interest earned when $t_1 + t_2 \le t_d$.

Cost of interest charges per year can be obtained as follow:

Interest payable per year = $C_i = 0$ (31)

Interest earned per year can be obtained as follow:

Interest earn per year =

$$C_e = i_e \sum_{j=1}^{n} \left[h(t_r) \int_0^{t_1} (d \cdot t) dt + H_{Ro} \int_{t_1}^{t_1+t_2} (d \cdot t) dt + \frac{t_1 H_{Ro} + t_2 h(t_r)}{t_1 + t_2} \left(t_d - (t_1 + t_2)\right) \int_0^{t_1+t_2} (d) dt \right]$$

(32)

The distributor's inventory cost during the planning horizon *T*, TC_R, is the sum of the ordering cost *ORC*, the holding cost HC_R^{Own} and HC_R^{Rent}, the deteriorating cost DC_R, the backordering cost *OC*, the interest earn and the interest payable. One has,

$$TC_R(q,n) = ORC + HC_R^{Own} + HC_R^{\mathrm{Rent}} + DC_R + OC + C_i - C_e \quad (33)$$

3.3 JOINT COST STRUCTURE OF SINGLE-SUPPLIER SINGLE-DISTRIBUTOR SYSTEM

The joint cost of the supplier and the distributor, *TC*, is the sum of TC_M and TC_R. The optimization problem of minimizing *TC* is a constrained nonlinear programming stated as:

$$\text{Minimize} \quad TC(q,n) = TC_M(q,n) + TC_R(q,n) \quad (34)$$

$$\text{Subject to} \quad w < q$$

$$0 \le t_1, t_2, t_3$$

$$0 < n$$

$$d < p$$

Our objective is to minimize the joint cost. By taking the first derivation of *TC*(*q*,*n*) with respect to *q* and *n* setting the result to zero, one has

$$\frac{\partial TC}{\partial q} = 0 \ \text{ and } \ \frac{\partial TC}{\partial n} = 0 \quad (35)$$

Eq.(33) can be solved simultaneously for *q** and *n** using Maple.

3.4 SOLUTION PROCEDURES

The heuristic iterative method is applied to solve the model with the use of Maple. The solution procedures are as follows:

Step 1. Input all the system parameters.

Step 2 Since the number of delivery, *n*, is an integer value, we start by choosing an integer value of $n \ge 1$.

Step 3. By taking the first derivative of $TC(q,n)$ with respect to *q* and setting the result to zero, the optimal solution (q^*, n^*) can be derived.

Step 4. Repeat Step 2 and Step 3. The optimal values, q^* and n^*, must satisfy the following condition:

$$TC(q(n^*-1), n^*-1) \geq TC(q^*, n^*) \leq TC(q(n^*+1), n^*+1)$$

Step 5. End

4. NUMERICAL EXAMPLE

The preceding theory can be illustrated by the numerical example extended from Yu (2007). The related input parameters for the supplier are: p=60 units per day, C_S=\$1000 per set-up, P_M=\$6 per unit, H_M=\$1 per unit per day, θ_1=0.05 and u=3. The related input parameters for the distributor are: d=45 units per day, $C_{R=}$\$1500 per order, C_o=\$9 per units, H_{Ro}=\$0.8 per unit per day, H_{Rr}=\$1 per unit per day, P_R=\$7 per unit, θ_2=0.045, θ_3=0.055, w=60 storage capacity of OW, i_c=0.02, i_e=0.01, t_r=30 days, $t_{\max}$=90 days, and t_d=0 days, 7 days, 30 days and 90 days respectively.

The optimal solution is: q^*= 296 units and n^*=50 deliveries; the admissible time periods are: t_1^*=4.71 days, t_2^*=1.00 days, t_3^*=1.59 days and t_p^*=5.67 days and the minimum joint cost is \$231,752. Since TC is a very complicated function due to the high-power expression of the exponential function, a graphical representation showing the convexity of the TC function is given in Fig. 6. From the set of parameters, it is seen that TC is strictly convex.

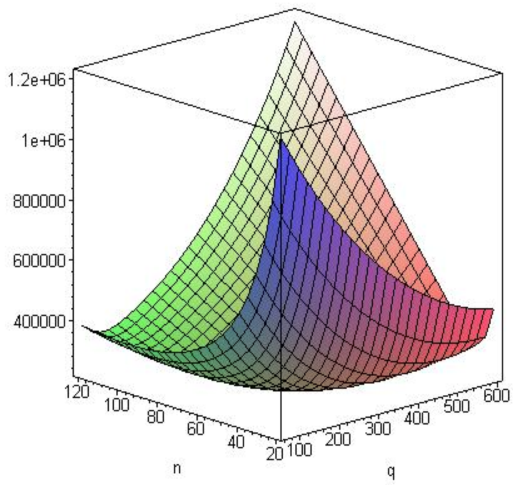

Fig. 6. Graphical representation of a convex TC (where q^*=296 and n^*=50)

In Table 1, it is shown that the integrated policy offers the lower total cost than when the members independently decide their lot size. It is clearly seen that if both the supplier and distributor agree on the optimal transportation frequency of n^* deliveries, the integrated system has a total cost saving of 67.3% or $155,976 from the supplier's perspective and 22.9% or $52,998 from the distributor's perspective. The reason is that the supplier prefers to deliver a large batch to reduce the setup cost. However, the total joint cost increases due to the increasing delivery frequency.

Table 1. Total joint cost for different policies (t_d = 90)

Policy	The optimal number of delivery	The optimal Ordering quantity	Total joint cost $($TC$)	*PICC*(%)
Supplier's perspective	8	1961	387,728	67.3
Distributor's perspective	39	513	284,750	22.9
Integrated policy	50	296	231,752	0

Note: TC^* is the global optimum of the integrated total cost *TC*.

PICC: Percentage of Integrated Cost Change = $(TC- TC^*)/TC^*$

As depicted in Table 2, the optimal number of delivery and total joint cost will decrease when the trade credit period increases. The cost saving is 8.19% as the trade credit period is from t_d = 0 days to t_d = 90 days.

Table 2. The minimum of the total joint cost for different trade credit period

The trade credit period	The optimal number of delivery	The optimal Ordering quantity	Total joint cost $($TC^*$)	*PICC*(%)
0	54	260	252,446	0
7	54	260	248,531	-1.55
30	52	277	239,890	-4.97
90	50	296	231,752	-8.19

Note: $TC^*_{t_d=0}$ is the global optimum of the integrated total cost.

PICC: Percentage of Integrated Cost Change = $(TC^*_{t_d=other}\text{-}TC^*_{t_d=0})/TC^*_{t_d=0}$

4.1 SENSITIVITY ANALYSES

With the integrated policy, the optimal values of q, n, t_p, t_1, t_2, t_3 and TC for a fixed set of parameters $\Phi=\{\ C_S,\ P_M,\ H_M,\ \theta_1,\ u,\ p, d,\ C_R,\ C_o,\ H_{Ro},$

H_{Rr} , P_R , θ_2 , θ_3 , w , i_c , i_e } are denoted by q^*, n^*, t_p^*, t_1^*, t_2^*, t_3^* and TC^*, respectively. Their changes are then considered when the parameters in the set Φ vary. A sensitivity analysis where each parameter in the set Φ increases or decreases by {-20%，-10%，0，+10%，+20%} is carried out. Only one parameter is changed and the other parameters remain constant. Table 3 shows the sensitivity analysis of *PICC* (Percentage of Integrated Cost Change) respectively when the parameter *d* is changed (t_d = 90). The PICC is achieved by (*TC*- *TC**)/*TC** where *TC* is the total joint cost when one parameter is changed and *TC** is the global optimum joint cost.

Table 3. The sensitivity analysis of *PICC* for the parameter *d* (t_d = 90)

	q^*	n^*	t_P	t_1	t_2	t_3	TC
−20%	290	43	5.53	5.62	1.18	1.69	198700
−10%	293	47	5.60	5.06	1.08	1.63	217214
0	296	50	5.67	4.71	1.00	1.59	231752
10%	297	54	5.69	4.30	0.92	1.54	249602
20%	299	57	5.73	4.04	0.87	1.50	263661

The major conclusions draw from the sensitivity analysis are as follows:

(1) When the distributor’s demand rate increases, each player’s total cost TC_M and TC_R, and the joint cost *TC* will increase.

(2) When the production rate increases, the value of *PICC* decreases and the frequency of delivery decreases to counteract the delivery cost.

(3) Among the deterioration rates, the value of *PICC* is most sensitive to the deterioration rate of the supplierθ_1.

(4) The value of *PICC* is most sensitive to *d* which is the parameter of the basic administration fee in RW. The value of *PICC* increases more than 14% when *d* increases by 20%.

(5) The parameters C_S , P_M , H_M , θ_1 , d , C_R , C_o , H_{Ro} , H_{Rr} , P_R , θ_2 , θ_3 , w , i_c influence the value of *PICC* in the same direction. The parameters *u*, *p* and i_e influence the value of *PICC* in the opposite direction.

(6) The PICC’s sensitivity related to the parameters in Φ can be ranked as:

-13% ~ +13%]: *d*;

-5% ~ +5%: *p, C_s, C_r*;

+1% ~ +3%: C_o , w, θ_1, θ_2, θ_3, H_M , P_M, P_R, i_c ;
≦1%: H_{Ro}, H_{Rr}.

5. CONCLUSION

This study developed a JELS policy for deteriorating items in a two-echelon system with decreasing RW storage price. The results showed that the total cost was less using the integrated policy than when the suppliers and the distributors determined their lot sizes independently. This implies that coordination among supply chain members is necessary for reducing total costs. In addition, decreasing the warehouse rental or increasing the trade credit will also reduce the total joint cost. This study provides managerial insight for enterprises that use an RW with a decreasing storage price. Using this study, enterprises can develop an optimal ordering policy by coordinating their lot sizes and trade credit to utilize the decreasing storage prices of RWs over time.

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