A two-stage supply contract with options, risky supplier and forecast updating

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Abstract

We develop a single product two-stage supply contract where a retailer buys a number of supply options from a main supplier at the beginning of the first decision stage (first period). The retailer faces a stochastic demand concentrated in the second stage (second period) and is modeled using a joint probability distribution with an exogenous information. In addition, the retailer has a second supply option from a risky supplier whose availability is modeled using a binomial distribution. At the beginning of the second decision stage, the stochastic exogenous information is revealed and the demand forecast is updated conditionally knowing the value of the exogenous information. Moreover, at the beginning of the second decision stage, the information about the availability or unavailability of the risky supplier becomes known with certainty. Therefore, the supply options bought from the main supplier can be transformed at the beginning of the second decision stage fully or partially into orders and delivered immediately. Moreover, if the risky supplier is available, another quantity may be ordered and delivered immediately from this supplier. The end customer demands occur during the second stage and every satisfied demand is charged a certain price by the retailer. At the end of the selling season, any remaining units are salvaged by the retailer at a salvage value.

We model this problem using a dynamic programming approach and we exhibit some characteristics of the structure of the optimal policy for the retailer for both available supply options. We provide the structure of the second decision stage optimal policy and some analytical insights concerning the first stage optimal policy. Furthermore, through a numerical study, we analyze the effect of some of the model parameters on the optimal policy especially the information quality, the probability of the availability of the risky supplier, the difference in the costs of the two supply options and the other economic parameters.

Keywords: inventory control, dual supply, risky supplier, options, forecast updating, short life-cycle products.

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1 Introduction

Newsvendor-type products are seasonal products with end customer demand that is concentrated in a short selling season. These products are usually perishable, which means that at the end of the selling season, any remaining items are salvaged at a salvage value that is less than the original price that is charged to the customers. Many of these products, especially the goods, are manufactured by an organization, the supplier, and sold to end-customers by another organization, the retailer in a decentralized decision-making context. Moreover, in nowadays business, decentralization is a fact in many supply chains for different reasons. For instance, outsourcing the production to independent production units automatically shifts the the decision making authority (Tsay, 1999). In the current global environment, another example of decentralization may happen within the same organization where different production sites report to different functional entities, maybe for incentive considerations. In this context, because of the manufacturing and logistics lead-times, the replenishment decisions are made by the retailer before the beginning of the selling season. Therefore, the replenishment decisions are made before the retailer has an accurate estimation of the demand, which leads to the use of forecasts. The high inaccuracy of the forecasts or the uncertain nature of the demand makes it better to use probability distributions to model the future end customer demand. In inventory management, the most suitable model to determine the stocking levels for this category of products is the well known *Newsvendor* model (Khouja, 1999). In this single period-model, the retailer orders a quantity at the beginning of the selling season at a certain unit csot, which is delivered immediately in order to satisfy the uncertain end-customer demand. During the selling season, any satisfied demand is charged a unit price and any unsatisfied demand is lost and a lost sales cost is incurred. At the end of the selling season, any remaining units are salvaged at a salvage value. Many extensions have been proposed in the literature to improve the *Newsvendor* model (Khouja, 1999). Some of these extensions model the problem using a two-period framework which allows the decision maker to react to any change in the demand during the first period. Other extensions include the use of some information updating mechanisms allowing to improve the quality of the demand forecasts using either indigenous or exogenous information. Indigenous information may represent information about the actual demand of the same product in previous selling periods. Exogenous information may be collected, for instance, by sales representatives from the distribution of sales vouchers or quotations, or in e-business from the number of visits to a commercial website. For instance, a visit to a certain subpage, the "wish lists" completed by the webpage visitors about products of interest, or the incomplete shopping carts could indicate the interest of the buyer by a specific product (Cheaitou et al., 2014).

Another *Newsvendor*-type inventory management framework that allows a better coordination between the supplier and the retailer is the supply contract. A supplier contract is an agreement between two parties in the supply chain, usually a supplier and a retailer or a distributer, in order to organize and optimize the production decisions of the supplier and the replenishment decisions of the retailer. Different types of supply contracts exist including the quantity-flexibility contracts, the backup contracts, the buy back contracts and the option-future contracts (Cheaitou et al., 2010). The difference between these different types of contracts lies mainly in the structure of the decision making process. Another differentiation aspect is the degree of coordination that these contracts may result in between the two contracting parties of the supply chain.

In this work we propose a supply contract model in which two decision periods are considered. At the beginning of the the first period, the retailer buys from the main supplier a number of supply options in order to satisfy end-customer demands which are concentrated in the second period and modeled using a probability distribution, while the information about the availability of another secondary supplier is random and modeled using Bernoulli distribution. At the beginning of the second period the availability of the secondary supplier is either confirmed or not and the demand probability distribution is updated using an exogenous information. The retailer may then transform part or the totality of the supply options into orders which are delivered immediately. If the secondary supplier is available then the retailer can order another quantity that is delivered immediately. At the end of the selling season the remaining units are salvaged by the retailer.

The remaining of this paper is structured as follows. First, the pertinent literature is reviewed which leads to develop our proposed model and define the optimization problem and the solution approach. In the following section, we present the numerical study in which we show the effect of some of the model parameters on the optimal policy. The last section is dedicated to the conclusion and to future research avenues.

2 Literature review

We divide the literature related to our work into three categories: *Newsvendor* inventory models with information updating, inventory models with dual sourcing and supply contract models.

The improvement of demand forecasts using updating processes in the context of inventory management has been investigated since the 1950s (Scarf, 1959; Murray and Silver, 1966; Azoury, 1985; Lovejoy, 1990). Two classes of forecast updating models exist. In the first class of models, an exogenous information is used to update the demand forecast. In particular, Gurnani and Tang (1999) consider a two-period model with no demand at the first period. At the end of the first period, and after a first procurement decision, exogenous information is collected, permitting to update the initial forecast for the second period demand. They investigate a special case in which the value of the information used to update the demand forecast varies from worthless to perfect. They model the unit ordering cost of the second period using a probability distribution with a value that can be higher or lower than the unit ordering cost at the first period. Moreover, Choi, et al. (2003) propose a similar two-period model where the second-period demand is updated using some market exogenous information. Yang et al. (2011) model a component-purchasing problem for a supply chain consisting of one retailer and two complementary suppliers with different lead-times using dynamic programming. After ordering from the long-lead-time supplier and before ordering from the short-lead-time supplier, the retailer can update its demand forecast for the product using a market signal. Cheaitou et al. (2014) develop a two-period inventory management model where the demand of both periods is stochastic. At the end of the first period, the demand of the second period is updated using exogenous information. They include in their model two sources of supply: one fast and one slow suppliers. The investigate the structure of the optimal policy of both periods. In the second class of demand forecast updating models, the first period demand is used as endogenous information to update the second period demand assuming that a correlation exists between the two demands. Many of the studies of this class are related to the quick response inventory management policy in the apparel industry (Fisher et al., 1994; Fisher et al, 2001; Choi et al., 2006). For example, Fisher and Raman (1996) modeled the demand of the whole horizon and the demand of the first period using a joint probability density function. They approximate the optimal numerically using heuristics approach. Tan et al. (2009) use advanced demand information to solve an inventory problem with two demand classes. Bradford and Sugrue (1990) use a Bayesian updating process to improve the quality of the second period demand using the observed the value of the first period demand. Ma et al.'s (2012) proposed a model in which the retailer has two ordering opportunities before demand is realized. In their model a forecast updating process is considered. Zheng et al. (2016) investigate an extension of the newsvendor model with demand forecast updating under supply constraints. In their model, a retailer can postpone order placement to improve the quality of the demand forecast while shortening the supply lead time. In this case, the supplier charges the retailer a higher cost set restrictions on the ordering times and quantities. They include in their model two supply modes: a supply mode that has a limited ordering time scale, and another one that has a decreasing maximum ordering quantity. For their demand forecasting process, they use the Martingale model of forecast evolution (MMFE).

In the category of literature that deals with the dual sourcing question, it is worth noting that the pioneering work was the model developed by Daniel (1963)) that focused on emergency shipments limited to two modes. Fukuda (1964) extended this work to include unbound emergency shipments and general lead time values. Whittemore and Saunders (1977) considered a general model with arbitrary lead times and identified cases in which it is optimal to use only one supply mode. Moinzadeh and Nahmias (1988) examined the basic dual supply problem in a continuous review setting. Zhang (1995) proposed a periodic review system that included up to three supply modes. Lawson (1995) considered a specific form of lead-time flexibility that is formally modeled as a series of expedite and de-expedite opportunities. More recently, Li et al. (2009) proposed a model with two procurement opportunities with lead times where the second order timing is s decision variable. Allon and Van Mieghem (2010) formulated a model in which two sourcing options are available: a responsive nearshore source and a low-cost offshore source with random demand. Boute and Mieghem (2011) analyzed a global dual sourcing policy with two suppliers: a responsive and expensive supplier and a slow and less costly supplier. They proposed a sourcing and ordering policy that allocates the order volume to both sources by finding an optimal trade off between cost and responsiveness. Other studies investigated the dual sourcing problem with different contexts such as minimum cumulative commitment and capacity (Xu, 2011), risk management (Xanthopoulos et al., 2012; Giri, 2011), the exponential utility of profit (Oberlaender, 2011) or lead-time reduction (Ryu and Lee, 2003). (For a detailed survey of this literature, we refer readers to Minner, 2003; Thomas and Tyworth, 2006; Jain et al., 2011; or Cheaitou and van Delft, 2013).

The third category of related works deals with supply contracts. As mentioned earlier, many types of supply contracts exist. The main differentiation aspect between them is the structure of the contract itself. The first type of contracts is the backup contract (Eppen and Iyer, 1997) that is characterized by an initial order that is made at a first decision point. At the final decision point, part of the initial order can be canceled, up to a certain predefined percentage. The second type of contracts is the options-futures contract ((Barnes-Schuster et al., 2002) and (Cachon and Lariviere, 2001)). This type of contracts is characterized by two decision periods. In the first period two decisions are made: the number of futures (a non-refundable and unchangeable commitment) and the number of options (a flexible commitment). In the second period part of the totality of the prescribed options can be transformed into orders by paying an exercise cost to the supplier. A third type of contracts is the quantity flexibility contract ((Bassok and Anupindi, 1995) and (Tsay, 1999)) in which an initial order is made and can later be revised within a certain range. The fourth type of contracts to mention here is the buy-back contract (Arshinder et al., 2009): any remaining units at the end of the selling season are returned to the supplier at a salvage value that represents a fraction of unit ordering cost. Other types of contracts exists such as the contracts with promotional efforts (Yan and Zaric, 2016), the contracts with revenue-sharing option (Arani et al., 2016) and the contracts with multiple service levels (Protopappa-Sieke et al., 2016).

It is worth noting that the forecast updating processes are widely used in the context of supply contracts (see (Bassok and Anupindi, 1995), (Barnes-Schuster et al., 2002), (Eppen and Iyer, 1997), (Tsay, 1999), and (Brown, 1999)).

To the best of our knowledge, none of the reviewed papers considered a two stage supply contact in which an external information is used to update the second period demand and two supply options are available: a main supplier with options and a secondary risky supplier. Therefore, this work contributes to the literature by especially allowing to investigate the effect of the existence of the risky supplier and the quality of the exogenous information on the ordering policy.

3 Model description

We consider a single product two-stage model where a retailer orders at the beginning of the first stage a number of options, K, at a unit price c_0, from a main supplier. These options represent capacity booking decisions that may be transformed into orders at the beginning of the second stage in order to satisfy a stochastic demand D. This random demand follows a joint probability distribution with an exogenous information ξ, that may represent some external information collected about the market during the first stage. Moreover, at the beginning of the first stage, the available information about the risky supplier may be summarized in a Bernoulli distribution where the supplier is supposed to be available at the beginning of the second stage with a probability of u and unavailable with a probability of $1-u$.

Let $\psi(\xi, D)$ be the joint probability density function (PDF) of the exogenous information and the demand, and let $\Psi(\xi, D)$ be their joint cumulative distribution function (CDF). The marginal CDF and PDF of the information ξ are denoted as $g(\xi)$ and $G(\xi)$ respectively, the conditional PDF and CDF of the demand D for any given value of ξ are $f(D|\xi)$ and $F(D|\xi)$ respectively.

At the beginning of the second decision stage, the exogenous information ξ is revealed and the demand distribution is updated conditionally to the value of ξ. Moreover, the availability of the risky supplier is either confirmed or not. The retailer can therefore transform the options ordered at the beginning of the first stage into orders, by ordering a quantity Q_1 at a unit cost of c_1, where $Q_1 \leq K$. Moreover, if the risky supplier is available, the retailer has also the possibility to order an unconstrained quantity Q_2 at a unit cost of c_2.

Furthermore, during the selling season (second period), every satisfied end-customer demand is charged at a price p and every unsatisfied demand is lost and a penalty shortage cost b is incurred. At the end of selling season, any remaining units are salvaged at a salvage value s.

4 Model assumptions

To avoid some trivial or non-realistic cases it is necessary to introduce some assumptions for the different parameters of the model. These assumptions are listed below:

$$p > c_0 + c_1, \ p > c_2 \tag{1}$$

$$s < c_0 + c_1, \ s < c_2 \tag{2}$$

Assumptions (1) are used to avoid unrealistic cases in which the customer is charged a price less than the ordering cost which leads to a pure loss scenario. Assumptions (2) are used to avoid cases in which the salvage value at the end of the selling season is greater than the ordering cost.

Furthermore, without loss of generality, we assume that the demand distribution is characterized as follows: the joint probability density function of the information ξ and the demand $D-$namely, $\psi(\xi, D)$ $-$is a bivariate normal distribution with means θ and μ, standard deviations δ and σ, and a correlation coefficient ρ. Therefore, the joint probability density function is of the form

$$\psi(z_\xi, z_D) = \frac{e^{-1/2\sqrt{1-\rho^2}(z_\xi^2 + z_D^2 - 2z_\xi z_D)}}{2\pi\delta\sigma}, \tag{3}$$

where

$$z_\xi = \frac{\xi - \theta}{\delta} \quad \text{and} \quad z_D = \frac{D - \mu}{\sigma}. \tag{4}$$

Furthermore, we know that the conditional demand $(D|\xi)$ is normally distributed with PDF $f(\cdot)$, CDF $F(\cdot)$, mean μ', and standard deviation σ' (Bickel and Doksum, 1977), where

$$\mu' = \mu + \rho\frac{(\xi - \theta)\sigma}{\delta} \quad \text{and} \quad \sigma' = \sigma\sqrt{1 - \rho^2}. \tag{5}$$

It is worth noting that any probability distribution other that the bivariate normal distribution may be used. The choice of one probability distribution, instead of keeping the general form $\psi(\xi, D)$, is a means to develop the optimization model, as it will be shown in the next sections.

4.1 The optimization problem

We define $\Pi_2^u(Q_1, Q_2|\xi)$ as the expected profit of the second period conditional to the information ξ, for the case in which the risky supplier is available and $\Pi_2^{1-u}(Q_1|\xi)$ for the case in which the risky supplier is not available. We introduce also $\Pi_1(K, Q_1, Q_2)$ as the expected profit of the first period. These expected profits are expressed as follows:

$$
\begin{aligned}
\Pi_2^u(Q_1, Q_2|\xi) = \quad & p \quad \int_0^{Q_1+Q_2} D\, f(D|\xi) dD + p(Q_1+Q_2) \int_{Q_1+Q_2}^{\infty} f(D|\xi) dD \\
+ \quad & s \int_0^{Q_1+Q_2} (Q_1+Q_2-D) f(D|\xi) dD - c_1 Q_1 - c_2 Q_2 \\
- \quad & b \int_{Q_1+Q_2}^{\infty} (D - Q_1 - Q_2) f(D|\xi) dD
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\Pi_2^{1-u}(Q_1|\xi) = \quad & p \quad \int_0^{Q_1} D\, f(D|\xi) dD + pQ_1 \int_{Q_1}^{\infty} f(D|\xi) dD \\
+ \quad & s \int_0^{Q_1} (Q_1 - D) f(D|\xi) dD - c_1 Q_1 \\
- \quad & b \int_{Q_1}^{\infty} (D - Q_1) f(D|\xi) dD
\end{aligned} \tag{7}
$$

$$
\Pi_1(K, Q_1, Q_2) = -c_0 K + E_\xi[u \Pi_2^u(Q_1, Q_2|\xi) + (1-u)\Pi_2^{1-u}(Q_1|\xi)] \tag{8}
$$

where $E_\xi[\cdot]$ represents the expected value with respect to the information ξ.

The optimization problem is then defined as:

$$
\max_{0 \le K, 0 \le Q_1 \le K, 0 \le Q_2} \Pi_1(K, Q_1, Q_2). \tag{9}
$$

5 The solution approach

In order to determine the optimal ordering policy, a two-stage stochastic dynamic programming approach is adopted where the optimization problem defined in (9) is decomposed into two coupled subproblems. First, the second stage subproblem is formulated and solved to determine the optimal values of the decision variable Q_1 and Q_2. Second, using the optimal policy of the second stage, the first stage subproblem is formulated and solved.

5.1 Second-period subproblem

In this section, we exhibit the solution of the second stage subproblem. Based on the availability of the risky supplier, two possible formulations, and therefore subproblems, are possible. The first formulation is as follows:

$$
\max_{0 \le Q_1 \le K, 0 \le Q_2} \Pi_2^u(Q_1, Q_2|\xi), \tag{10}
$$

and correspond to the case in which the risky supplier is available and the second formulation is as follows:

$$
\max_{0 \le Q_1 \le K} \Pi_2^{1-u}(Q_1|\xi), \tag{11}
$$

and corresponds to the case in which the risky supplier is unavailable. We prove the concavity of the expected objective function and then we develop the optimal ordering policy for both cases.

5.2 Case 1: available risky supplier

Lemma 1 *The objective function $\Pi_2^u(Q_1, Q_2|\xi)$ defined in (6) is a jointly concave function with respect to Q_1 and Q_2.*

Proof 1 *The Hessian of $\Pi_2^u(Q_1, Q_2|\xi)$ with respect to Q_1 and Q_2 is given by*
$\nabla^2\Pi_2^u(Q_1, Q_2|\xi) = -(p + b - s)f(Q_1 + Q_2|\xi) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
For each vector

$$V = (V_1; V_2)$$

where $(V_1; V_2) \in \mathbb{R}^2$, we find

$$V^T\nabla^2\Pi_2^u(Q_1, Q_2|\xi)V = -(p + b - s)f(Q_1 + Q_2)(V_1 + V_2)^2.$$

By assumption (2), one has $s < p$ which means that $(p + b - s) > 0$. Hence, we have

$$V^T\nabla^2\Pi_2^u(Q_1, Q_2|\xi)V \leq 0$$

which proves that the matrix $\nabla^2\Pi_2^u(Q_1, Q_2|\xi)$ is semi-definite negative. Consequently, the objective function $\Pi_2^u(Q_1, Q_2|\xi)$ is jointly concave with respect to Q_1 and Q_2, which completes the proof. □

Consider the two partial derivatives of $\Pi_2^u(Q_1, Q_2|\xi)$ with respect to Q_1 and Q_2, respectively, given by

$$\frac{\partial\Pi_2^u}{Q_1} = (p - c_1 + b) - (p + b - s)F(Q_1 + Q_2|\xi) \tag{12}$$

and

$$\frac{\partial\Pi_2^u}{Q_2} = (p - c_2 + b) - (p + b - s)F(Q_1 + Q_2|\xi) \tag{13}$$

Setting the first partial derivatives (12) and (13) equal to zero, we obtain

$$F(Q_1 + Q_2|\xi) = \frac{p - c_1 + b}{p - s + b} \tag{14}$$

$$F(Q_1 + Q_2|\xi) = \frac{p - c_2 + b}{p - s + b} \tag{15}$$

Hence, two threshold levels appear to be of great importance in the second period optimal policy characterization,

$$Y_1(\xi) = F^{-1}\left(\frac{p - c_1 + b}{p - s + b}|\xi\right) \tag{16}$$

$$Y_2(\xi) = F^{-1}\left(\frac{p - c_2 + b}{p - s + b}|\xi\right) \tag{17}$$

Taking into account the bivariate joint normal distribution of the demand and the information as it was detailed in (3)-(5), and for a given value if the information ξ, the thresholds can be rewritten as follows:

$$Y_1(\xi) = \mu + (\xi - \theta)\frac{\sigma}{\delta}\rho + (\sigma\sqrt{1 - \rho^2})\Phi^{-1}\left(\frac{p - c_1 + b}{p - s + b}\right), \tag{18}$$

and

$$Y_2(\xi) = \mu + (\xi - \theta)\frac{\sigma}{\delta}\rho + (\sigma\sqrt{1 - \rho^2})\Phi^{-1}\left(\frac{p - c_2 + b}{p - s + b}\right). \tag{19}$$

5.2.1 Structure of the optimal policy

In order to develop the optimal policy of the second period, we distinguish between two cases: $c_1 < c_2$ and $c_1 > c_2$.

Case 1.a: $c_1 < c_2$

Case 1.a.1: $K < Y_2(\xi)$

Lemma 2 *For $K < Y_2(\xi)$, the optimal solution is given by*

$$Q_1^* = K \text{ and } Q_2^* = Y_2(\xi) - K.$$

Proof 2 *For $K < Y_2(\xi)$, one finds*

$$\begin{aligned}
\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = K, Q_2^* = Y_2(\xi) - K) &= (p - c_1 + b) - (p - s + b)F(K + Y_2(\xi) - K|\xi) \\
&= (p - c_1 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\
&= (p - c_1 + b) - (p - s + b)\frac{p - c_2 + b}{p - s + b} \\
&= c_2 - c_1 > 0 \qquad (20)
\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = K, Q_2^* = Y_2(\xi) - K) &= (p - c_2 + b) - (p - s + b)F(K + Y_2(\xi) - K|\xi) \\ &= (p - c_2 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\ &= (p - c_2 + b) - (p - s + b)\frac{p - c_2 + b}{p - s + b} \\ &= 0 \end{aligned} \qquad (21)$$

which induces, by concavity, that the solution $Q_1^* = K$ and $Q_2^* = Y_2(\xi) - K$ is the optimum of the profit function for such K values. □

Case 1.a.2: $Y_2(\xi) < K < Y_1(\xi)$

Lemma 3 *For $Y_2(\xi) < K < Y_1(\xi)$, the optimal solution is given by*

$$Q_1^* = K \ \text{ and } \ Q_2^* = 0.$$

Proof 3 *For $Y_2(\xi) < K < Y_1(\xi)$, one finds from the monotonicity of $F(.)$*

$$\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = K, Q_2^* = 0) = (p - c_1 + b) - (p - s + b)F(K) > 0 \qquad (22)$$

and

$$\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = K, Q_2^* = 0) = (p - c_2 + b) - (p - s + b)F(K) < 0. \qquad (23)$$

Since $Q_1 < K$ and $Q_2 > 0$, we deduce, by concavity, that the solution $Q_1^* = K$ and $Q_2^* = 0$ is the optimum of the profit function. □

Case 1.a.3: $K > Y_1(\xi)$

Lemma 4 *For $K > Y_1(\xi)$, the optimal solution is given by*

$$Q_1^* = Y_1(\xi) \ \text{ and } \ Q_2^* = 0.$$

Proof 4 *For $K > Y_1(\xi)$, one finds*

$$\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = Y_1(\xi), Q_2^* = 0) = (p - c_1 + b) - (p - s + b)F(Y_1(\xi)) = 0 \qquad (24)$$

and

$$\begin{aligned}\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = Y_1(\xi), Q_2^* = 0) &= (p - c_2 + b) - (p - s + b)F(Y_1(\xi)) \\ &= (p - c_2 + b) - (p - s + b)\frac{p - c_1 + b}{p - s + b} \\ &= c_1 - c_2 < 0. \end{aligned} \qquad (25)$$

Thus, we deduce, by concavity, that the solution $Q_1^* = Y_1(\xi)$ and $Q_2^* = 0$ is the optimum of the profit function. □

Case 1.b: $c_1 > c_2$
Case 1.b.1: $K < Y_1(\xi)$

Lemma 5 *For $K < Y_1(\xi)$, the optimal solution is given by*

$$Q_1^* = 0 \text{ and } Q_2^* = Y_2(\xi).$$

Proof 5 *For $K < Y_1(\xi)$, one finds*

$$\begin{aligned}\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = 0, Q_2^* = Y_2(\xi)) &= (p - c_1 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\ &= (p - c_1 + b) - (p - s + b)F(Y_1(\xi)|\xi) \\ &= (p - c_1 + b) - (p - s + b)\frac{p - c_2 + b}{p - s + b} \\ &= c_2 - c_1 < 0 \end{aligned} \tag{26}$$

and

$$\begin{aligned}\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = 0, Q_2^* = Y_2(\xi)) &= (p - c_2 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\ &= (p - c_2 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\ &= (p - c_2 + b) - (p - s + b)\frac{p - c_2 + b}{p - s + b} \\ &= 0 \end{aligned} \tag{27}$$

which induces, by concavity, that the solution $Q_1^* = 0$ and $Q_2^* = Y_2(\xi)$ is the optimum of the profit function for such K values. □

Case 1.b.2: $Y_1(\xi) < K < Y_2(\xi)$

Lemma 6 *For $Y_1(\xi) < K < Y_2(\xi)$, the optimal solution is given by*

$$Q_1^* = 0 \text{ and } Q_2^* = Y_2(\xi).$$

Proof 6 *For $Y_1(\xi) < K < Y_2(\xi)$, one finds*

$$\begin{aligned}\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = 0, Q_2^* = Y_2(\xi)) &= (p - c_1 + b) - (p - s + b)F(Y_2(\xi)|\xi) \\ &= c_2 - c_1 < 0 \end{aligned} \tag{28}$$

and

$$\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = 0, Q_2^* = Y_2(\xi)) = (p - c_2 + b) - (p - s + b)F(Y_2(\xi)) = 0. \tag{29}$$

We deduce, by concavity, that the solution $Q_1^* = 0$ and $Q_2^* = Y_2(\xi)$ is the optimum of the profit function. □

Case 1.b.3: $K > Y_2(\xi)$

Lemma 7 *For $K > Y_2(\xi)$, the optimal solution is given by*

$$Q_1^* = 0 \text{ and } Q_2^* = Y_2(\xi).$$

Proof 7 *For $K > Y_2(\xi)$, one finds*

$$\frac{\partial \Pi_2^u}{\partial Q_1}(Q_1^* = 0, Q_2^* = Y_2(\xi)) = (p - c_1 + b) - (p - s + b)F(Y_2(\xi)) < 0 \tag{30}$$

and

$$\frac{\partial \Pi_2^u}{\partial Q_2}(Q_1^* = 0, Q_2^* = Y_2(\xi)) \;\; = \;\; (p - c_2 + b) - (p - s + b)F(Y_2(\xi)) = 0. \tag{31}$$

Thus, we deduce, by concavity, that the solution $Q_1^* = 0$ and $Q_2^* = Y_2(\xi)$ is the optimum of the profit function. □

As a conclusion on the optimal ordering policy of the retailer,if $c_1 < c_2$, the retailer uses only the main supplier except when the number of options K verifies $K < Y_2(\xi)$; in fact, in this case, the retailer orders from both the main supplier and the risky supplier. In addition, if $c_2 < c_1$, the risky supplier will be the only source of supply.

5.3 Case 2: unavailable risky supplier

Lemma 8 *The objective function $\Pi_2^{1-u}(Q_1|\xi)$ defined in (7) is a jointly concave function with respect to Q_1.*

Proof 8 *The Hessian of $\Pi_2^{1-u}(Q_1|\xi)$ with respect to Q_1 is given by*
$\nabla^2 \Pi_2^u(Q_1|\xi) = -(p + b - s)f(Q_1)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
For each vector

$$V = (V_1; V_2)$$

where $(V_1\,; V_2) \in R^2$, we find using assumption (2)

$$V^T \nabla^2 \Pi_2^{1-u}(Q_1|\xi)V = -(p + b - s)f(Q_1)(V_1 + V_2)^2 \leq 0$$

which proves that the matrix $\nabla^2 \Pi_2^{1-u}(Q_1, Q_2|\xi)$ is semi-definite negative. Consequently, the objective function $\Pi_2^{1-u}(Q_1|\xi)$ is jointly concave with respect to Q_1. □

5.3.1 Structure of the optimal policy

Lemma 9 *For $K > Y_1(\xi)$, the optimal solution is given by*

$$Q_1^* = Y_1(\xi).$$

Proof 9 *Setting the partial derivative of $\Pi_2^{1-u}(Q_1)$ with respect to Q_1 equal to zero yields*

$$F(Q_1|\xi) = \frac{p - c_1 + b}{p - s + b} = 0.$$

Thus, for $K > Y_1(\xi)$, the optimal solution is given by

$$Q_1^* = F^{-1}\left(\frac{p - c_1 + b}{p - s + b}\right).$$

which completes the proof. □

Lemma 10 *For $K < Y_1(\xi)$, the optimal solution is given by*

$$Q_1^* = K.$$

Proof 10 *The partial derivative of $\Pi_2^{1-u}(Q_1)$ with respect to Q_1 is given by*

$$\frac{\partial \Pi_2^{1-u}}{\partial Q_1}(Q_1) = (p - c_1 + b) - (p + b - s)F(Q_1)$$

Hence, since $K < Y_1(\xi)$, it yields using the monotonicity of $F(.)$ that

$$\frac{\partial \Pi_2^{1-u}}{\partial Q_1}(Q_1^* = K) = (p - c_1 + b) - (p + b - s)F(K) > 0.$$

Thus, $Q_1^* = K$ is the optimum of the profit function. □

5.4 First-period subproblem

The second period optimal policy that was obtained earlier will be used to solve the first period subproblem, which can be obtained by substituting the decision variables Q_1 and Q_2 by their respective optimal values Q_1^* and Q_2^* in the optimization problem defined in (9) which leads to:

$$\max_{0 \leq K} \Pi_1(K, Q_1^*, Q_2^*). \tag{32}$$

where

$$
\begin{aligned}
\Pi_1(K,Q_1^*,Q_2^*) = & - c_0K + \int_0^{K\frac{\delta}{\rho\sigma}-\alpha_1} \left(p\int_0^{Y_1(\xi)} D\, f(D|\xi)dD - c_1Y_1(\xi) \right. \\
& + pY_1(\xi)\int_{Y_1(\xi)}^{\infty} f(D|\xi)dD + s\int_0^{Y_1(\xi)} (Y_1(\xi)-D)\, f(D|\xi)dD \\
& \left. - b\int_{Y_1(\xi)}^{\infty} (D-Y_1(\xi))f(D|\xi)dD \right) g(\xi)d\xi \\
& + u\left[\int_{K\frac{\delta}{\rho\sigma}-\alpha_1}^{K\frac{\delta}{\rho\sigma}-\alpha_2} \left(p\int_0^{K} D\, f(D|\xi)dD + pK\int_K^{\infty} f(D|\xi)dD \right.\right. \\
& \left. + s\int_0^{K} (K-D)\, f(D|\xi)dD - b\int_K^{\infty} (D-K)f(D|\xi)dD - c_1K \right) g(\xi)d\xi \\
& + \int_{K\frac{\delta}{\rho\sigma}-\alpha_2}^{\infty} \left(p\int_0^{Y_2(\xi)} D\, f(D|\xi)dD + pY_2(\xi)\int_{Y_2(\xi)}^{\infty} f(D|\xi)dD \right. \\
& + s\int_0^{Y_2(\xi)} (Y_2(\xi)-D)\, f(D|\xi)dD - c_1K - c_2(Y_2(\xi)-K) \\
& \left.\left. - b\int_{Y_2(\xi)}^{\infty} (D-Y_2(\xi))f(D|\xi)dD \right) g(\xi)d\xi \right] \\
& + (1-u)\left[\int_{K\frac{\delta}{\rho\sigma}-\alpha_1}^{\infty} \left(p\int_0^{K} D\, f(D|\xi)dD + pK\int_K^{\infty} f(D|\xi)dD \right.\right. \\
& \left.\left. + s\int_0^{K} (K-D)\, f(D|\xi)dD - b\int_K^{\infty} (D-K)f(D|\xi)dD - c_1K \right) g(\xi)d\xi \right]
\end{aligned} \tag{33}
$$

where

$$
\alpha_1 = \frac{\mu\delta}{\sigma\rho} + \frac{\delta}{\rho}\sqrt{1-\rho^2}\Phi^{-1}\left(\frac{p-c_1+b}{p-s+b}\right) - \theta, \tag{34}
$$

and

$$
\alpha_2 = \frac{\mu\delta}{\sigma\rho} + \frac{\delta}{\rho}\sqrt{1-\rho^2}\Phi^{-1}\left(\frac{p-c_2+b}{p-s+b}\right) - \theta, \tag{35}
$$

Lemma 11 *labelfirstperiodobjconcavlemma The first period expected objective function defined in (33) is concave with respect to the decision variable K.*

Using the concavity of the first period optimal policy, the optimization model defined in (32) and (33) will be solved numerically using the numerical optimization function *NMaximize* of the software *Mathematica 10.2* on a Windows 7 Entreprise 64-bit platform equipped with an Intel Core i7-4810MQ 2.8GHz 2.8GHz CPU and 16 GB of RAM. The results are reported in Section **??**.

6 A special case: the perfect information setting

In this section, we develop the optimal policy when the correlation between exogenous information and demand is perfect (i.e. $\rho = 1$). The other special case whereby $\rho = 0$ is obvious and will be discussed in numerical analysis.

When $\rho = 1$, the conditional demand distribution becomes deterministic where the conditional demand value is equal to $D|\xi = \mu + \frac{(\xi-\theta)\sigma}{\delta}$. The thresholds defined in (18) and (19) become as follows:

$$Y_1(\xi) = Y_2(\xi) = \mu + (\xi - \theta)\frac{\sigma}{\delta}. \tag{36}$$

It is worth noting that $D|\xi = Y_1(\xi)$. It is also worth noting that α_1 becomes equal to α_2.

6.1 Second period optimal policy

The second period optimal policy has been completely characterized in Section 5.

Therefore, for the perfect information case and for the values of $\xi < K\frac{\delta}{\rho\sigma} - \alpha_1$ one has $D|\xi < K$ which means that, for the second period optimal policy, three cases are to be considered:

- the risky supplier is available:
  - if $c_1 < c_2$, then $Q_1^* = D|\xi$ and $Q_2^* = 0$
  - if $c_1 > c_2$, then $Q_1^* = 0$ and $Q_2^* = D|\xi$
- the risky supplier is unavailable:
  - $Q_1^* = D|\xi$ and $Q_2^* = 0$

For the values of $\xi > K\frac{\delta}{\rho\sigma} - \alpha_1$ one has $D|\xi > K$ which means that, for the second period optimal policy, three cases are also to be considered:

- the risky supplier is available:
  - if $c_1 < c_2$, then $Q_1^* = K$ and $Q_2^* = D|\xi - K$
  - if $c_1 > c_2$, then $Q_1^* = 0$ and $Q_2^* = D|\xi$
- the risky supplier is unavailable:
  - $Q_1^* = K$ and $Q_2^* = 0$

6.2 First period optimal policy

For the first period, by rearranging the terms and using Equation (36), the expected objective function defined in (33) becomes as follows:

$$\begin{aligned}
&\Pi_1(K, Q_1^*, Q_2^*) = \qquad (37)\\
-\quad & c_0 K + \int_0^{\frac{\delta(K-\mu)}{\rho\sigma}+\theta} \left(p \int_0^{Y_1(\xi)} Y_1(\xi)\, f(D|\xi) dD - c_1 Y_1(\xi) + p Y_1(\xi) \int_{Y_1(\xi)}^{\infty} f(D|\xi) dD \right) g(\xi) d\xi \\
+\quad & (1-u) \int_{\frac{\delta(K-\mu)}{\rho\sigma}+\theta}^{\infty} \left((p-s+b) \int_0^K (Y_1(\xi) - K)\, f(D|\xi) dD + (p - c_1 + b)K - b\mu \right) g(\xi) d\xi
\end{aligned}$$

7 Conclusion

In this paper we developed a two-period mathematical model for supply contract design with options and demand forecast updating that governs the ordering process of a retailer from two suppliers: a main supplier and a secondary risky supplier. A certain number of options are ordered from the main supplier at the beginning of the first period and can be transformed into orders at the beginning of the second period. The retailer has to satisfy a stochastic end-customer demand that is concentrated in the second period. During the first period, an exogenous information is collected and used to update the demand forecast at the beginning of the second period. Moreover, the availability of the risky supplier is modeled using Bernoulli distribution. At the beginning of the second period, the availability of this supplier is either confirmed or not. If the risky supplier turns out to be available, then an unconstrained quantity may be ordered from this supplier. Any satisfied end-customer demand is charged a unit price and any unsatisfied demand is lost and a penalty shortage cost is incurred. At the end of the selling season any remaining units are salvaged at a salvage value. We modeled this problem using stochastic dynamic programming. We obtained the optimal ordering policy of the retailer for the second period and we showed that the first period objective function is concave with respect to the number of options. We studied a special case in which the quality of the exogenous information is perfect which means that the correlation between the exogenous information and the demand is complete. We presented some numerical applications that focused on the effect of some of the model parameters on the optimal policy and the objective function. Characterizing analytically the first period optimal policy or considering stochastic costs for the risky supplier in the second period may be avenues for future research.

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